Models of Computation

Tutorial Exercises 3

1. The alphbet is $\Sigma = \{ 0, 1 \}$. Give context-free grammars that generate the following languages.

   (i) The empty set.

   (ii) The language of palindomes.

   (iii) The set of strings containing at least three 1's.

   (iv) The set of odd-length strings whose middle symbol is 0.

2. Find context-free grammars for the following languages (where $m, n \geq 0$):

   (a) $L = \{ a^n b^m : n \leq m + 3 \}$

   (b) $L = \{ a^n b^m : n \neq 2m \}$

3. (a) Describe what is meant by *well-bracketing* in the case where there are two kinds of parentheses, namely, ( ) and [ ]. For example ([ ]) and ([ ] (( ))) [ ] are well-bracketed, but ([ )] is not.

   Give a CFG over $\Sigma = \{ [,], (,) \}$ that generates well-bracketed parentheses.

   (b) Say that (and] are positive and) and [are negative. A string over Σ is said to be *alternating* if any two adjacent symbols have opposite polarities.

   Give a CFG that generates the set L of alternating and well-bracketed strings over Σ. For example [( )] [ ] and ([([ ])]) ( ) are in L but [ ] ( ) and [[ ]] are not.

4. Show that the following grammars are ambiguous by exhibiting distinct leftmost derivations:

   (a) $(\{ S \}, \{ a, b \}, \mathcal{R}, S)$ with $\mathcal{R} = \{ \; S \rightarrow a\,S\,b\,S \mid b\,S\,a\,S \mid \epsilon \; \}$

   (b) $(\{ S, A, B \}, \{ a, b \}, \mathcal{R}, S)$ with $\mathcal{R} = \{ \; S \rightarrow A\,B \mid a\,a\,B, \quad A \rightarrow a \mid A\,a, \quad B \rightarrow b \; \}$

5. Let L be the set of binary strings w such that every postfix of w has at least as many 0's as 1's. A *postfix* of a string w is any v such that $w = u\,v$ for some u. For example $01100, 1010 \in L$ but $110 \notin L$.

   (a) Find a CFG for L, and show that it is correct. Is your grammar ambiguous?

   (b) [*Harder*] If not done as answer for (a) find an *unambiguous* grammar for L, and justify your answer fully.

6. Give an implementation-level NPDA that accepts the language generated by the CFG $(\{ S, A \}, \{ a, b \}, \mathcal{R}, S)$ with $\mathcal{R} = \{ \; S \rightarrow A\,A \mid a, \quad A \rightarrow S\,A \mid b \; \}$

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