

MODELS OF COMPUTATION

Tutorial Exercises 1

1. The alphabet is $\{0, 1\}$. Give DFAs that accept the following languages:
 - (i) The empty set.
 - (ii) The singleton set consisting of the empty string.
 - (iii) All strings except the empty string.
 - (iv) $\{w : w \text{ starts with } 0 \text{ and has odd length, or starts with } 1 \text{ and has even length}\}$
 - (v) $\{w : w \text{ contains at least two } 0\text{s and at most one } 1\}$
 - (vi) $\{w : w \text{ contains an even number of } 0\text{s, or exactly two } 1\text{s}\}$
2. Let L_1 and L_2 be regular languages over the alphabet Σ .
 - (i) Using the product construction on DFAs, give a detailed proof that $L_1 \cap L_2$ is regular.
 - (ii) Prove that $L_1 \setminus L_2 \stackrel{\text{def}}{=} \{w \in L_1 : w \notin L_2\}$ is regular.
3. The alphabet is $\{0, 1\}$. Give NFAs with the specified number of states accepting the following languages:
 - (i) The language $\{w : w \text{ ends with } 00\}$ with three states.
 - (ii) The language $\{\epsilon\}$ with one state.
 - (iii) The language $\{0\}$ with two states.
 - (iv) All words that start and end with the same symbol, with four states.
4. The alphabet is $\{0, 1, \dots, 9\}$. Give NFAs that accept the following languages:
 - (i) The set of strings such that the final digit has appeared before.
 - (ii) The set of strings such that the final digit has *not* appeared before.
5. Design NFAs for the following languages.
 - (i) The set of strings consisting of zero or more a 's, followed by zero or more b 's, followed by zero or more c 's.
 - (ii) The set of strings of 0's and 1's such that at least one of the last ten positions is a 1.
6.
 - (i) Let M be an NFA that accepts language L . Does swapping the final and non-final states in M necessarily yield an NFA? Does the new NFA so obtained necessarily accept the complement of L ?
 - (ii) Are regular languages closed under complement? Justify your answer.
7. Give regular expressions for each of the following subsets of $\{a, b\}^*$.
 - (i) $\{x : x \text{ contains an even number of } a\text{'s}\}$
 - (ii) $\{x : x \text{ contains an odd number of } b\text{'s}\}$
 - (iii) $\{x : x \text{ contains an even number of } a\text{'s or an odd number of } b\text{'s}\}$

8. Give NFAs accepting the sets of strings denoted by the following regular expressions:

(i) $(000^* + 111^*)^*$

(ii) $(01 + 10)(01 + 10)(01 + 10)$

(iii) $(0 + 1(01^*0)^*1)^*$

Try to simplify as much as possible.

HN, Hilary 2004