Convergence of AFEM for General Elliptic Operators and the Laplace-Beltrami Operator

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We first review the basic principles for convergence of adaptive finite element methods (AFEM) for the Laplace operator, as shown recently by P. Morin, R.H. Nochetto, and K. Siebert. Marking according to a posteriori error estimators and data oscillation, followed by refinement with an interior node property, guarantees energy and data oscillation reduction and leads to convergence of adaptive loops with a linear rate. This result hinges on an orthogonality property for the energy norm as well as the crucial fact that energy error and data oscillation decouple for the Laplace operator. This is no longer true for general elliptic operators with variable coefficients and the Laplace-Beltrami operator on surfaces. In both cases the energy error couples with either the approximation of coefficients or the surface (geometric error). We next present novel error/oscillation and error/geometry reduction estimates, based on quasi-orthogonality properties, which show that AFEM are a contraction for their collective contribution. We also illustrate the theory with numerical experiments, which lead to optimal meshes. These results are joint with K. Mekchay and P. Morin.