hp–Adaptive Discontinuous Galerkin Methods with Interior Penalty for Degenerate Elliptic Partial Differential Equations

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Abstract

We consider the *a posteriori* and *a priori* error analysis of the *hp*–version of the discontinuous Galerkin finite element method with interior penalty for approximating second–order partial differential equations with nonnegative characteristic form. In particular, we discuss the question of error estimation for certain linear target functionals of the solution of practical interest; relevant examples include the local mean value of the field or its flux through the outflow boundary of the computational domain Ω, and the evaluation of the solution at a given point in Ω.

Our *a posteriori* error bounds stem from a duality argument and include computable residual terms multiplied by local weights involving the solution of a certain *dual* or *adjoint* problem. Guided by our *a posteriori* error analysis, we design and implement an adaptive finite element algorithm to ensure reliable and efficient control of the error in the computed functional with respect to a user–defined tolerance. A key question in *hp*–adaptive algorithms is how to automatically decide when to *h*–refine/derefine and when to *p*–refine/derefine. To this end, we construct an appropriate adaptive strategy based on estimating the local Sobolev regularities of the primal and dual solutions. The performance of the resulting *hp*–refinement algorithm is demonstrated through a series of numerical experiments. In particular, we demonstrate the superiority of using *hp*–adaptive mesh refinement with the traditional *h*–refinement method, where the degree of the approximating polynomial is kept fixed at some low value.

References

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