

Convergence Rates for Adaptive Finite Element Methods

Peter Binev, Wolfgang Dahmen, and Ronald DeVore

University of South Carolina

RWTH Aachen

University of South Carolina

Adaptive Finite Element Methods (AFEM) are often used in practice but it is only recently that they have been shown to converge. This paper will introduce an AFEM based on piecewise linears on partitions generated by newest vertex bisection for solving Laplace's equation on a polygonal domain in two dimensions, and show that it converges at an optimal rate in the following sense. If the solution u can be approximated (assuming complete knowledge of u) to accuracy $O(n^{-s})$, $s > 0$, $n = 1, 2, \dots$, by piecewise linear functions on a partition with n triangles generated by newest vertex bisection, then the adaptive algorithm will provide the same rate of approximation while using only information gained through computations executed in the algorithm. The algorithm is a modification of ideas of Doerfler, and Morin, Nochetto, and Siebert. The main new ingredient is the addition of a coarsening step. The analysis is based on the theory of nonlinear approximation. We also uncover several new interesting results about newest vertex bisection and adaptive strategies.