

Towards multilevel meshless methods

Wolfgang Dahmen^a, Shai Dekel^b, and Pencho Petrushev^c

^aInstitut für Geometrie und Praktische Mathematik, RWTH Aachen, 52056 Aachen, Germany

^bDepartment of Mathematics, University of Tel Aviv, Tel Aviv, Israel

^cDepartment of Mathematics, University of South Carolina, Columbia, SC 29208, U.S.A.

A scale of nonlinear smoothness spaces is presented that is capable among other things to capture anisotropic features. It can be based on multiscale collections of atoms that are products of polynomials and smooth compactly supported cutoff functions forming on each scale a partition of unity. It is indicated under which circumstances representations in terms of such atoms characterize the above mentioned smoothness classes and related best N -term approximation. Possible applications concern meshless methods for the numerical treatment of elliptic boundary value problems based on such collections of atoms. Possible ways of deriving preconditioners from such norm equivalences are discussed that work, in particular, for adaptive approximations.